# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2965

AN ANALYSIS OF NORMAL-ACCELERATION AND AIRSPEED DATA FROM
A FOUR-ENGINE TYPE OF TRANSPORT AIRPLANE IN COMMERCIAL
OPERATION ON AN EASTERN UNITED STATES ROUTE
FROM NOVEMBER 1947 TO FEBRUARY 1950

By Thomas L. Coleman and Paul W. J. Schumacher

Langley Aeronautical Laboratory
Langley Field, Va.



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# SUMMARY

An analysis of 48,187 hours of normal-acceleration and airspeed data obtained on a four-engine type of transport airplane in commercial operation on an eastern United States route from November 1947 to February 1950 has been made to determine the gusts and gust loads for the operations. The results, covering operations to 20,000 feet, indicate that the maximum gust velocity encountered in a given number of flight miles was less than that for a low-altitude operation (below 10,000 feet) but the accelerations experienced were about as severe in terms of percentage of limit load factor as for the operations below 10,000 feet. A somewhat greater frequency of attaining high airspeeds for these operations than for some other previously reported transport operations is also indicated. The gusts and gust loads during the summer (Apr. through Sept.) were approximately 20 percent more severe than during the winter (Oct. through Mar.). The gusts encountered for the operations were about the same as those encountered for airplanes of the same type operated by the British on the North Atlantic route.

# INTRODUCTION

In the study of the gusts and gust loads for commercial transport airplanes, normal-acceleration and airspeed data have been collected over a number of years from NACA V-G recorders installed in airplanes of various types (ref. 1). In the past these data have been used in the formulation of gust-load design requirements for transport airplanes. Consequently, the data have been collected with the aim of obtaining representative samples of the gusts and gust loads for transport operations on trunk airlines covering various climatic and terrain conditions within the United States and on oceanic routes. Most of the data obtained to date have been collected from operations generally below an altitude of 10,000 feet. The first data from higher altitudes were reported in reference 2, which presented an analysis of a limited sample of data (9300 hr covering less than a year of operations) from a modern four-engine type

of transport airplane operating to altitudes of about 20,000 feet on a north-south route over the eastern United States. Record collection for these operations has been completed and the total data sample covers about 50,000 hours of flight during a period of over two years

An analysis of these data is presented in this paper. Estimates are given of the frequency of occurrence of given values of acceleration, gust velocity, and maximum airspeeds. Effects of the airspeed practice and of the seasons on the gusts and gust loads are indicated. Comparisons are made between the present results and the results previously obtained from other operations.

# SYMBOLS

A	aspect ratio
K	gust alleviation factor (ref. 3)
$P_{\Delta n}$	probability that maximum acceleration increment on record will equal or exceed given value
$P_{\mathbf{v}}$	probability that maximum indicated airspeed on record will equal or exceed given value
<sup>P</sup> Ue	probability that maximum effective gust velocity on record will equal or exceed given value
U <sub>e</sub>	effective gust velocity, fps
U <sub>emax</sub>	maximum effective gust velocity, fps
<sup>U</sup> e30	effective gust velocity of 30 fps
$v_{L}$	design level flight speed (indicated), mph
$v_{\text{max}}$	maximum indicated airspeed on record, mph
$V_{O}$	indicated airspeed at which maximum positive or negative acceleration increment occurs on record, mph
$V_p$	probable indicated airspeed at which maximum acceleration increment will occur, mph
$v_{ m NE}$	placard never-exceed indicated airspeed, mph

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$\Delta n_{max}$	maximum positive or negative acceleration increment on record, g units
∆n <sub>LLF</sub>	acceleration increment corresponding to limit-gust-load-factor increment for gross weight, g units
α	statistical scale parameter of distribution of extreme values (ref. 4, p.2)
u	location parameter of distribution of extreme values (ref. 4, p.2)
ន	best small-sample estimate of standard deviation of distribution of extreme values (ref. 4, p.2)
k	coefficient of skewness of distribution (called $\alpha$ in ref. 5)
σ	standard deviation for distribution of variable (ref. 5)

A bar over a symbol denotes mean value of distribution.

# APPARATUS AND SCOPE OF DATA

The data were obtained by means of two types of NACA V-G recorders: the friction-damped recorder (ref. 6) and the oil-damped recorder (ref. 7). Both recorders are similar except for the method of damping the accelerometer element. The precision of the two types of recorders is discussed in reference 7. Inherent instrument errors are assumed to be less than ±0.2g for acceleration and about 3 percent of the maximum airspeed range for the friction-damped recorders; whereas the corresponding errors for the oil-damped recorders are less than ±0.1g for acceleration and about 1 percent of the maximum airspeed range. Approximately 60 percent of the data were obtained with the oil-damped recorders.

The data were collected on 10 four-engine airplanes engaged in commercial transport (passenger) operations on a north-south route in the area of the United States east of the Mississippi River. The length of flights (airline distance between stops) varied from about 100 miles to 1200 miles and the scheduled time for the flights varied from 1/2 hour to 5 hours. Detailed information on the altitudes at which the flights were made is not available. The operator, however, indicated that most of the flight time was spent at altitudes between 10,000 and 20,000 feet. As no detailed information was provided on actual operating weights, a value of 85 percent of the design gross weight, which is assumed as a representative average, was taken as the weight at which the gusts were encountered.

Characteristics of the airplanes on which the data were collected are given in table I and were obtained from the manufacturer's design data or the operator unless otherwise referenced. The limit gust load factor of 2.48g was computed from the gust-load-factor formula in reference 3 and is based on the maximum take-off weight, the computed lift-curve slope as recommended in reference 8, and on an effective gust velocity  $U_e$  of 30K feet per second at the maximum speed in level flight  $V_L$  of 271 miles per hour. The computed lift-curve slope of 4.93 used to obtain uniformity in evaluations is slightly higher than the slope of 4.67 which was used in design. As has been done in previous gust-load evaluations, the maximum gross weight was used to compute the limit gust load factor, although the use of an average operating weight would have yielded a slightly higher factor possibly more representative of the operations.

A total of 223 records representing 50,830 flight hours were available for analysis. In order to keep the range in flight hours per record small, however, only records representing between 200 and 250 flight hours were used. Consequently, the analysis utilized 194 records representing a total of 48,187 flight hours. Included in these records are the 38 records previously analyzed in reference 2.

#### RESULTS

Presentation of data .- From each record the maximum positive and negative acceleration increment  $\Delta n_{max}$ , the two speeds corresponding to the occurrence of the maximum acceleration increment  $V_0$ , the maximum positive and negative effective gust velocities  $U_{e_{\max}}$ , and the maximum airspeed  $V_{\text{max}}$  were determined. In addition, the maximum positive and negative accelerations in each 20-mile-per-hour airspeed bracket were read. The assumption is made that the maximum accelerations above 160 miles per hour were caused by gusts. No values were read from the records below this airspeed in order to avoid the possible inclusion of acceleration due to maneuvers and landing shock. Since it has been noted on time-history records that only about 2 percent of the flight time is spent at airspeeds below 160 miles per hour, the amount of data lost by this procedure is small. The effective gust velocities obtained were computed with the sharp-edged-gust equation (ref. 3) by using the computed slope of the lift curve and the assumed average operating weight of 79,900 pounds (0.85 gross weight) with the corresponding K value.

The frequency distributions of  $\Delta n_{max}$ ,  $V_0$ ,  $V_{max}$ , and  $U_{emax}$  for the data sample are given in table II grouped in class intervals of 0.lg, 10 miles per hour, 5 miles per hour, and 2 feet per second, respectively.

The distributions of  $\Delta n_{max}$  by airspeed bracket are given in table III. In order to smooth out irregularities in the data and to provide a mathematical description of the distributions, theoretical curves were fitted to each distribution given in table II. Curves based on the theory of extreme values (ref. 4) were used for the distributions of  $\Delta n_{max}$  and  ${\rm U_{e_{max}}}$  and Pearson type III curves (ref. 5) for the distribution of  ${\rm V_{0}}$ and  $V_{\text{max}}$ . The parameters defining each fitted curve are included in table II with the appropriate distribution. From previous experience these curves generally have been found to provide a good description of the data. The value of  $V_{\text{D}}$  given in table II was computed from the parameters of the curve for the Vo distribution by using the equation for the mode given in reference 9, page 92. The curves fitted to the distributions of  $\Delta n_{\text{max}}\text{, }Ue_{\text{max}}\text{, }\text{and }V_{\text{max}}\text{ were transformed to curves}$ showing the average flight miles required to equal or exceed given values of the variables by multiplying  $1/P_{\Delta n}$ ,  $1/P_{U_e}$ , and  $1/P_V$  by an assumed cruising speed of 217 miles per hour (0.8 $V_{\rm L}$ ) and the average flight hours per record, 248 hours. Inasmuch as the analysis is concerned mainly with the high values of  $\Delta n_{\text{max}}$ ,  $U_{\text{emax}}$ , and  $V_{\text{max}}$  recorded, only the parts of the transformed curves for the larger values of the variables are given in figures 1 to 4. The cumulative data points for each distribution are shown in the proper figure in order to show goodness of fit of the curves to the data.

Previous investigations generally have assumed the distributions of maximum positive and negative acceleration increments to be similar except for sign. Inspection of figure 1 indicates that the two distributions are practically identical for this data sample. Consequently, the frequency of occurrence of either positive or negative acceleration increments may be represented by the combined  $\Delta n_{\rm max}$  distribution as in figure 2.

Effect of changes in operating conditions. During the period covered by the data, two changes in operating conditions were made which possibly influenced the maximum airspeeds obtained. First, the operator, after noting the frequency with which high values of airspeed were obtained, stressed to the pilots the desirability of avoiding excessive airspeeds. Second, at about the same time a different type of propeller (which was placarded for certain ranges of revolutions per minute) was installed on the airplanes. In order to determine whether these two changes had any effect on the maximum airspeeds attained, Pearson type III probability curves were fitted to the distribution of Vmax for the periods before and after the changes. The two curves are given on a flight-miles basis in figure 5 with the cumulative data points of the distributions.

Effect of seasons. - In order to determine whether any seasonal variation existed in the accelerations, gusts, and airspeeds, the data

were grouped into two seasons: summer, April through September, and winter, October through March. Records which were not taken wholly within either season were omitted from the seasonal analysis. The frequency distributions of  $\Delta n_{\rm max},~V_{\rm o},~V_{\rm max},$  and  $U_{\rm e_{max}}$  for the two seasons are given in table IV with the appropriate parameters of each distribution. The transformed curves for the  $\Delta n_{\rm max},~U_{\rm e_{max}},$  and  $V_{\rm max}$  distributions are given in figures 6, 7, and 8, respectively, with the cumulative data points for each distribution.

Comparison with other results.— In order to compare the gusts and accelerations for the present operations with results obtained from previous investigations (refs. 10 and 11), the flight miles to exceed  $\Delta n_{\rm LLF}$  and  $U_{\rm e}_{\rm 30}$  for the operations are summarized in table V. Reference 10 did not contain effective gust velocities for the two operations reported therein; consequently, the flight miles to exceed  $U_{\rm e}_{\rm 30}$  for these operations were computed from the original data covered by the reference. The ratio of the most probable speed for maximum acceleration occurrence to the design maximum level flight speed  $V_{\rm p}/V_{\rm L}$  for the various operations is also included in table V.

Data on the loads experienced during British operations of the same type of airplane on North Atlantic routes have been presented in reference 12. In order to compare the gusts encountered on eastern United States operations with those encountered on the trans-Atlantic operations, flight envelopes of effective gust velocities for the two operations have been calculated and are presented in figure 9. The envelopes give the boundaries which in 10,000 hours of operation are expected to be exceeded, on the average, by one positive and one negative gust velocity independent of speed and by one maximum airspeed. The envelopes were calculated essentially in accordance with the method used in reference 12, the primary differences being that the positive and negative Δn<sub>max</sub> values were combined without regard to sign to form a single distribution for each airspeed bracket and extreme-value curves were fitted to these distributions instead of Pearson type III curves. The data used to compute the envelope for the present operations were the distributions of  $\Delta n_{max}$ given in table III and the distribution of  $V_{max}$  given in table II. The data for the operations on the North Atlantic route were taken from tables 1 to 3 of reference 12. The one airspeed and two acceleration values noted in the tables as being "suspect" were not used in calculating the envelope. The gust velocities are based on the same quantities as were used previously, except that a weight of 82,500 pounds (given as average operating weight in ref. 12) was used in conjunction with the British data as compared with the assumed average operating weight of 79,900 pounds used with the present data.

Reliability .- In order to determine the reliability of the estimates of the flight miles to exceed  $\Delta n_{\rm LLF}$  and  $U_{e_{30}}$ , confidence bands (ref. 13) were fitted to the  $\triangle n_{\rm LLF}$  and  $U_{\rm e_{max}}$ distributions. The confidence bands provide a measure of the range within which, for a given probability level (a probability level of 95 percent is used herein), the true value may be expected to lie. The results indicated that 95 percent of the estimates for the flight miles to exceed  $\Delta n_{T,T,F}$  would lie within a range from 0.7 to 1.4 times the estimated value given in figure 2. The estimates of the flight miles to exceed  $\Delta n_{\rm LLF}$  for the summer and winter operations, being based on smaller data samples, were indicated to be reliable within a range from about 0.6 to 1.6 times the estimated values given in figure 6. Similar ranges of reliability were indicated for the estimate of the flight miles to exceed  $U_{e_{30}}$  for the total and summer and winter operations given in figures 3 and 7. No adequate method is, at present, available for determining the reliability of estimates of the flight miles to exceed VNE. Since the data sample is quite large and no extrapolation of the data was required, the estimated flight miles to exceed VNE is felt to be reliable within a factor of about 2.

Although it is recognized that dynamic response of the airplane in gusts can have appreciable effect on accelerations recorded at the center of gravity of the airplane, these effects for the present data are not known. The dynamic response of the present airplane, however, is expected to be of the same order of magnitude as that for other present-day airplanes. Consequently, comparisons such as are made in table V of the flight miles to exceed  $\Delta n_{\rm LLF}$  and  $U_{\rm e30}$  for the present airplane and for airplanes of similar design should not be seriously in error. The data for the present operations and for the North Atlantic operations reported in reference 12 should be directly comparable since they were obtained on the same type of airplane. (See fig. 9 for the comparison of the gust velocities of these two operations.)

### DISCUSSION

Accelerations.— Examination of figure 2 indicates that, for the present data covering operations to an altitude of about 20,000 feet, the acceleration increment corresponding to the calculated limit-gust-load-factor increment of 1.48g may be exceeded, on the average, twice in about 1.1  $\times$  10<sup>6</sup> flight miles. For the other operations summarized in table V which generally were below an altitude of 10,000 feet, the distance required to exceed  $\Delta n_{LLF}$  twice varies from 1.2  $\times$  10<sup>6</sup> flight miles for the twin-engine airplane on the northern-transcontinental operations to 5.6  $\times$  10<sup>6</sup> and 2,000  $\times$  10<sup>6</sup> flight miles for the four-engine airplane

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operated on the Caribbean—South American and trans-Pacific routes, respectively. Thus, in terms of the percentage of limit load factor, the accelerations experienced during the present operations appear to be on the same level as those for the northern-transcontinental operations and somewhat higher than those for the Caribbean—South American operations. In comparison with the trans-Pacific operations, the accelerations for the present operations were significantly more severe.

Consideration of figure 6 indicates that, for a given mileage level, the acceleration increments obtained during the summer (Apr. through Sept.) were approximately 20 percent larger than those obtained during the winter (Oct. through Mar.). Since available information indicates that the average operating weights during the two seasons were about the same, the difference in accelerations must have resulted from differences in the gust velocities or airspeeds.

Gusts.- Inspection of figure 3 indicates that for the present operations an effective gust velocity  $U_{\rm e}$  of 30 feet per second may be encountered, on the average, twice in about  $0.5\times10^6$  flight miles. For operation of the twin-engine airplane on the northern transcontinental route the distance to exceed  $U_{\rm e}_{\rm 30}$  is about  $0.2\times10^6$  flight miles (table V). The corresponding values for the trans-Pacific and Caribbean—South American operations are given in table V as  $6\times10^6$  and  $0.8\times10^6$  flight miles, respectively. On the basis of the distance to exceed  $U_{\rm e}_{\rm 30}$ , therefore, the gusts encountered during the present operations appear to be slightly less severe than those for the northern-transcontinental operations but somewhat more severe than for the Caribbean—South American operations. The gusts encountered during the present operations were significantly greater than those for the trans-Pacific operation.

Comparison of the envelopes of effective gust velocities in figure 9 indicates that the gust boundaries predicted for 10,000 hours of operation for the present and North Atlantic (British) operations are quite similar in shape and that the curve for the present operations predicts only slightly higher gust velocities. The gust velocities for the two operations therefore appears to be about equal.

Other investigations generally have indicated a reduction in the amount of turbulence with increasing altitude. That the gusts for the present operations to 20,000 feet were not appreciably less than for the lower-altitude operations was therefore unexpected. The reason that a significant reduction in the gusts was not obtained is not known. Since the gust velocities for the present and North Atlantic operations are about equal, the lack of a reduction in the occurrence of gusts probably is not due solely to the particular route. Although corrections for

dynamic response could alter the results, it is felt that the relative gust velocities for the operations would not be changed significantly.

Inspection of figure 7 shows that for a given mileage level the gusts encountered during the summer were roughly 20 percent higher than those encountered during the winter. The higher gust velocities during the summer were probably the predominant factor causing the high accelerations during this season. Although the summer was more turbulent than the winter for the present operations, the reverse was found to be true for operations on a northern transcontinental route (ref. 11). It would appear that the seasonal gust encounters depend upon the route and that no general conclusion regarding the relative turbulence for summer and winter operations can be made.

Airspeeds. The ratio of the most probable speed for  $\Delta n_{max}$  occurrence to the design level flight speed  $V_p/V_L$  for the present data is 0.83 as given in table V. The  $V_p/V_L$  ratios for other operations summarized in table V range from 0.71 to 0.75. The  $V_p/V_L$  ratio of 0.83 for the present operation is therefore higher than the ratios for the other operations. Thus, the relatively high speed at which the airplanes were operated in rough air may have been instrumental in giving higher accelerations for the present operations than were expected.

The most probable speed  $V_p$  for  $\Delta n_{max}$  encounter is given in table IV to be 220 and 231 miles per hour for the summer and winter seasons, respectively. Since the summer was found to be the most turbulent, the slightly lower  $V_p$  for this season may indicate a practice of reducing airspeed when flying in extremely rough air.

Consideration of the total distribution of maximum airspeeds shown in figure 4 indicates that the average distance to exceed the design never-exceed speed of 324 miles per hour is about 1.6  $\times$  106 flight miles. This estimate for the flight miles to exceed  $V_{\mathrm{NE}}$  is greater than the estimate of  $0.5 \times 10^6$  flight miles given in reference 2 based on a smaller data sample. The difference between the two estimates may be attributed to a general reduction in the maximum airspeeds obtained after the time the pilots were instructed on the desirability of avoiding high airspeeds and a different type of propeller was installed on the airplanes. Inspection of figure 5 indicates that, although the maximum airspeed obtained for each period was about the same, the maximum airspeeds attained after the instructions to the pilots and propeller change were, on the average, about 15 miles per hour less than those previously attained. If this reduction in average maximum airspeeds is assumed to continue for future operations, a better estimate of the flight miles to exceed VNE may be obtained from the  $V_{\text{max}}$  distribution for the period after the propeller

change and instructions to the pilots than from the total  $V_{\text{max}}$  distribution. Based on the distribution of  $V_{\text{max}}$  for the period after the changes, an estimate of  $3.8 \times 10^6$  flight miles to exceed  $V_{\text{NE}}$  is obtained (fig. 5).

The estimate of  $3.8\times10^6$  flight miles to exceed  $V_{\rm NE}$  lies at the lower end of the range of estimates for operations previously investigated. This result would indicate a somewhat greater tendency towards high airspeeds for the present operations than for other operations for which data are available.

Examination of figure 8 indicates that the difference in the maximum airspeeds for the summer and winter seasons is not significant.

# SUMMARY OF RESULTS

The gusts and accelerations for operation of a four-engine airplane on an eastern United States route are analyzed and presented. The results are compared with those for operations of a different type of four-engine airplane on a Caribbean—South American route and a trans-Pacific route, and for operations of a twin-engine transport airplane operating on a northern transcontinental route. The operations analyzed herein were at altitudes up to 20,000 feet, whereas the three other operations were generally below 10,000 feet.

The accelerations for the present operations were somewhat more severe than for the Caribbean—South American operations and significantly more severe than for the operations on the trans-Pacific route. They were on about the same level as those for the northern transcontinental route. The accelerations for the eastern United States operations were about 20 percent higher during the summer (Apr. through Sept.) than during the winter (Oct. through Mar.).

The average number of flight miles to encounter a gust velocity of 30 feet per second  $U_{\rm e30}$  is about the same as that for a similar airplane operated by the British on the North Atlantic route. The number of flight miles to encounter  $U_{\rm e30}$  is less than the distance required on the Caribbean—South American operation but greater than the distance for a northern-transcontinental operation. In comparison with the trans-Pacific operations, the present results show a significant reduction in the average flight miles to exceed  $U_{\rm e30}$ . The intensity of the gusts was roughly

20 percent higher during the summer than during the winter; this result being the reverse of that reported for the northern-transcontinental operations. In comparison with other operations the present airplanes were operated at relatively high speed in rough air.

The design never-exceed speed of 324 miles per hour may be exceeded, on the average, once in about  $3.8\times10^6$  flight miles which is somewhat more frequent than for operations previously investigated. For purposes of statistical interpretation, it is significant to note that changes in operating practices and airplane characteristics which resulted in a reduction of about 15 miles per hour in the average maximum airspeeds were made during the period covered by the data.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 21, 1952.

# REFERENCES

- 1. Walker, Walter G., and Steiner, Roy: Summary of Acceleration and Airspeed Data From Commercial Transport Airplanes During the Period From 1933 to 1945. NACA TN 2625, 1952.
- 2. Coleman, Thomas L.: An Analysis of the Normal Accelerations and Airspeeds of Several Lockheed Constellation L-649 Airplanes in Postwar Commercial Transport Operations Over the Eastern Part of the United States. NACA RM L9E18a, 1950.
- 3. Anon.: Airplane Airworthiness Transport Categories. Pt. 4b of Civil Air Regulations, Civil Aero. Board, U. S. Dept. Commerce, July 20, 1950.
- 4. Press, Harry: The Application of the Statistical Theory of Extreme Values to Gust-Load Problems. NACA Rep. 991, 1950. (Supersedes NACA TN 1926.)
- 5. Peiser, A. M., and Wilkerson, M.: A Method of Analysis of V-G Records From Transport Operations. NACA Rep. 807, 1945. (Supersedes NACA ARR L5J04.)
- 6. Rhode, Richard V.: Gust Loads on Airplanes. SAE Jour., vol. 40, no. 3, Mar. 1937, pp. 81-88.
- 7. Taback, Israel: The NACA Oil-Damped V-G Recorder. NACA TN 2194, 1950.
- 8. Pierce, Harold B., and Trauring, Mitchell: Gust-Tunnel Tests To Determine Influence of Airfoil Section Characteristics on Gust-Load Factors. NACA TN 1632, 1948.
- 9. Elderton, W. Palin: Frequency Curves and Correlation. Cambridge Univ. Press, 1938.
- 10. Coleman, Thomas L., and Schumacher, Paul W. J.: An Analysis of the Normal Accelerations and Airspeeds of a Four-Engine Airplane Type in Postwar Commercial Transport Operations on Trans-Pacific and Caribbean—South American Routes. NACA TN 2176, 1950.
- ll. Steiner, Roy: An Analysis of Normal Accelerations and Airspeeds of One Type of Twin-Engine Transport Airplane in Commercial Operations Over a Northern Transcontinental Route. NACA TN 2833, 1952.
- 12. Starkey, R. D.: Analysis of the Accelerations and Speeds Recorded on B.O.A.C. Constellation Aircraft. TN No. Structures 48, British R.A.E., Oct. 1949.

13. Kimball, Bradford F.: Sufficient Statistical Estimation Functions for the Parameters of the Distribution of Maximum Values. The Annals of Mathematical Statistics, vol. XVII, no. 3, Sept. 1946, pp. 299-309.

# TABLE I.- CHARACTERISTICS OF THE AIRPLANE

Maximum take-off weight, lb	94,000 1650
Wing span, ft	123 14.67
Aspect ratio	9.17
Slope of lift curve per radian, computed from $\frac{6A}{A+2}$	4.93
Limit gust load factor (computed), g units	2.48
For 94,000 pounds (gross weight)	1.204 1.190
Design maximum level flight speed (indicated), VL, mph	271
Design never-exceed speed (indicated), VNE, mph	324

TABLE II.- FREQUENCY DISTRIBUTIONS OF  $\Delta D_{max}$ ,  $V_O$ ,  $V_{max}$ , AND  $U_{e_{max}}$  FOR TOTAL DATA SAMPLE

Acceleration increments					Aireg	Gusts			
An <sub>max</sub> ,	Frequency		1	Yo, mph	o, mph Frequency		Frequency	Ucmax, fps	Frequency
B 0000	Δn	<u>∆n</u>	Total	<u> </u>					
0.3 to 0.4 .4 to .5 .5 to .6 .6 to .7 .7 to .8 .8 to .9 .9 to 1.0 1.1 to 1.2 1.2 to 1.3 1.3 to 1.5 1.5 to 1.6 1.6 to 1.7 1.7 to 1.8 1.8 to 1.9 1.9 to 2.1 2.1 to 2.2	2 1 12 15 27 37 30 28 15 16 3 2 1	14 13 13 14 12 15 15 15 16 17 17 18 11 18 11 15 16 17 17 18 18 18 19 19 19 19 19 19 19 19 19 19 19 19 19	2 5 25 34 6 4 5 5 6 2 1 4 6 3 5 5 5 3 1 1 1 1 1	160 to 170 170 to 180 180 to 190 190 to 200 200 to 210 210 to 220 220 to 230 230 to 240 240 to 250 250 to 260 260 to 270 270 to 280 280 to 290 290 to 300 300 to 310	18 25 16 26 49 33 52 43 40 23 7 1	270 to 275 275 to 280 280 to 285 285 to 290 290 to 295 295 to 300 300 to 305 305 to 310 310 to 315 315 to 320 320 to 325 325 to 330 330 to 335 335 to 340	1 10 25 42 32 27 19 21 4 6 1	8 to 10 12 14 16 18 22 24 26 28 33 24 35 38 49 44 44 48 50 50 50 50 50 50 50 50 50 50 50 50 50	3 4 21 35 55 48 34 8 15 9 4 8 37 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Total	194	194	388	Total	388	Total.	a <sub>193</sub>	Total	388
An <sub>mex</sub> B U a	0.944 0.276 0.820 4.64	0.944 0.292 0.813 4.38	0.944 0.284 0.817 4.51	V̄ <sub>o</sub> or k V̄ <sub>p</sub>	222.4 28.9 0.187 225.1	⊽ <sub>melx</sub> σ k	299.6 11.5 0.765	U <sub>emex</sub> s u c	21.6 6.321 18.73 0.203

A maximum indicated airspeed of 356 miles per hour from one record was not used in the analysis because, according to information furnished by the operator, it occurred during an emergency descent.

TABLE III.- FREQUENCY DISPRIBUTIONS OF  $\Delta c_{max}$  BY AIRSPEED BRACKETS

Δn <sub>max</sub> ,				Frequency dis	tribution for ai	rspeeds of -			
g units	160 to 180 mph	180 to 200	200 to 220 mph	220 to 240 mph	2 <sup>1</sup> 40 to 260 mg/b	260 to 280 mph	280 to 300 mg/h	300 to 320	320 to 340 mgh
0 to 0.05 .05 to .10 .10 to .15 .15 to .20 .20 to .25 .25 to .35 .35 to .40 .40 to .45 .45 to .50 .50 to .75 .75 to .80 .80 to .75 .75 to .80 .80 to .90 .90 to 1.05 .90 to 1.05 .10 to 1.15 .115 to 1.10 .105 to 1.10 .105 to 1.25 .125 to 1.40 .140 to 1.45 .155 to 1.40 .140 to 1.55 .155 to 1.65 .155 to 1.65 .155 to 1.70 .175 to 1.85 .1.85 to 1.40 .1.85 to 1.55 .1.95 to 1.65 .1.95 to 1.75 .1.75 to 1.80 .1.80 to 1.55 .1.95 to 1.90 .1.80 to 1.95 .1.95 to 1.95	2 5 8 199 299 394 396 14 6 8 5 3 1 3 3 1 1 1 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 1	11 12 12 12 12 12 12 12 12 12 12 12 12 1				1 1 3 7 8 21 37 4 50 3 34 50 3 34 50 3 34 50 5 7 7 4 1 1 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	17717 140 150 160 160 160 160 160 160 160 160 160 16	3 14 30 29 14 19 8 4 7 2 5 1 1	35410 3 111 11 11 11 11 11 11 11 11 11 11 11
2.00 to 2.05 2.05 to 2.10 2.10 to 2.15 2.15 to 2.20		- - 1	  	  	1  				
Total	388	388	388	388	388	387	373	141	20
Zm <sub>mex</sub> u u	0.649 0.187 0.565 6.86	0.706 0.229 0.603 5.61	0.741 0.234 0.635 5.46	0.759 0.255 0.644 5.03	0,708 0,255 0,593 5,02	0.611 0.202 0.520 6.34	0.392 0.179 0.311 7.16	0.232 0.142 0.168 9.01	0.216 0.135 0.151 8.05

Table iv.- frequency distributions of  $\Delta n_{\max}, \, v_o, \, \, v_{\max}, \, \text{and} \, \, u_{o_{\max}}$  by season

[Summer: Apr. through Sept.; winter: Oct. through Mar.]

Acceleration increments			``	Airs	abeeq				Gusts		
Δn <sub>max</sub> ,	Frequ	nency	W	Frequ	ency	T	Frequ	ency		Frequency	
g units	Summer	Winter	Vo, mph	Summer	Winter	V <sub>menax</sub> , mph	Summer	Winter	U <sub>Cmax</sub> , fps	Summer	Winter
0.3 to 0.4 .4 to .5 .5 to .6 .6 to .7 .7 to .8 .8 to .9 .9 to 1.0 1.0 to 1.1 1.1 to 1.2 1.2 to 1.3 1.3 to 1.4 1.4 to 1.5 1.5 to 1.6 1.6 to 1.7 1.7 to 1.8 1.8 to 1.9 1.9 to 2.0 2.0 to 2.1 2.1 to 2.2	1 1 8 5 1 29 22 27 9 11 6 5 2 2 2 4 1 1	1 35 28 35 28 17 8 4 1 1 3 1 1 1	160 to 170 170 to 180 180 to 190 180 to 200 200 to 210 210 to 230 230 to 240 240 to 250 250 to 260 260 to 270 270 to 280 280 to 290 290 to 300 300 to 310	4891218 218265118 4311 1184311	13 13 148 11 20 27 19 14 1 1	270 to 275 275 to 280 280 to 285 285 to 295 290 to 300 300 to 305 305 to 310 310 to 315 315 to 320 320 to 325 325 to 330 330 to 335 335 to 340	1 8 19 13 11 6 1 3 2 1	1 8 13 15 14 70 3 1 2	8 10 12 14 16 18 20 22 24 26 28 33 24 34 35 38 49 24 14 14 18 20 22 24 26 28 33 24 35 38 49 24 14 14 18 20 22 24 26 28 30 22 35 38 39 49 24 14 14 18 20 22 24 26 28 30 22 35 38 39 49 24 14 14 18 20 22 25 25 25 35 38 39 49 24 14 14 18 20 22 25 25 25 25 25 25 25 25 25 25 25 25	2 16 10 17 16 24 22 17 7 9 2 8 6 1 1	13223432217534 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Total	152	184	Total.	152	184	Total	76	92	Total	152	184
Zm <sub>meax</sub> s u c	1,01 0,307 0,875 4,17	0.87 0.247 0.759 5.18	∇o d k ∀p	220.9 27.1 0.0715 220.0	224.6 30.8 0.417 231.0	Ϋ́max σ k	301.6 11.5 0.928	298.6 11.7 0.699	Uemax u a	23.0 7.15 19.782 0.179	20.1 5.40 17.624 0.238

TABLE V.- SUMMARY OF FLIGHT MILES TO EXCEED  $\Delta n_{\rm LLF}$  AND  $u_{\rm e_{30}}$  FOR VARIOUS OPERATIONS Based on data uncorrected for dynamic response

Q	Advolene	$v_{\rm p}/v_{\rm L}$	Flight miles to exceed twice			
Operation	Airplane	at ∆n <sub>max</sub>	$\Delta n_{I,I,F}$	<sub>Ue30</sub>		
Present Trans-Pacific (ref. 10) Caribbean-South American	4-engine transport	0.83 .71	1.1 × 10 <sup>6</sup> 2000	0.5 × 10 <sup>6</sup>		
(ref. 10)	·	.72	5 <b>.</b> 6	0.8		
Northern transcontinental (ref. 11)	2-engine transport	•75	1.2	0.2		

NACA

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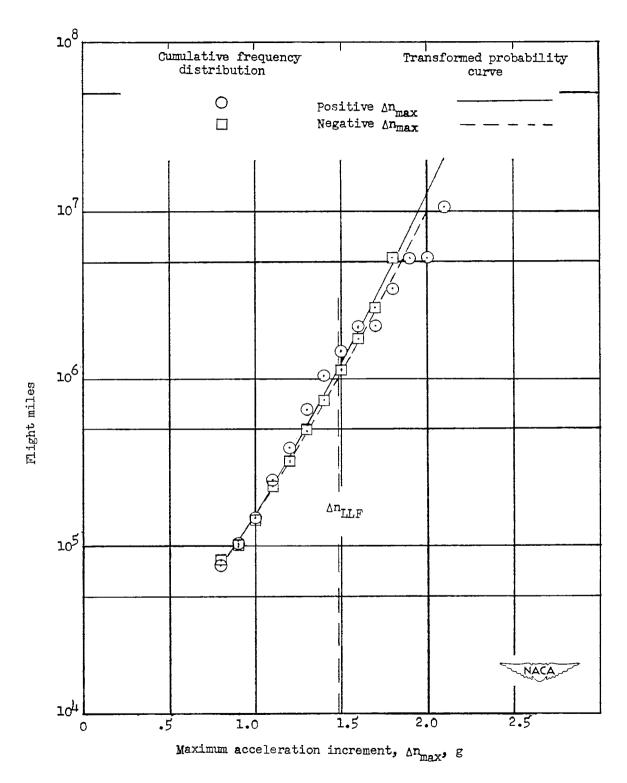


Figure 1.- Comparison of the average flight miles required for a positive and a negative acceleration increment to equal or exceed a given value.

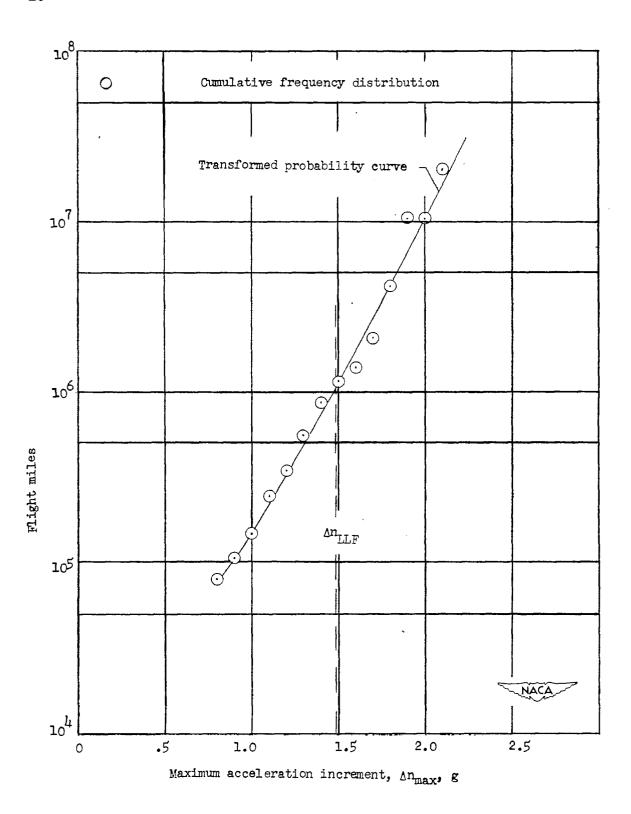


Figure 2.- Average flight miles required for an acceleration increment to equal or exceed a given value twice (positive and negative).

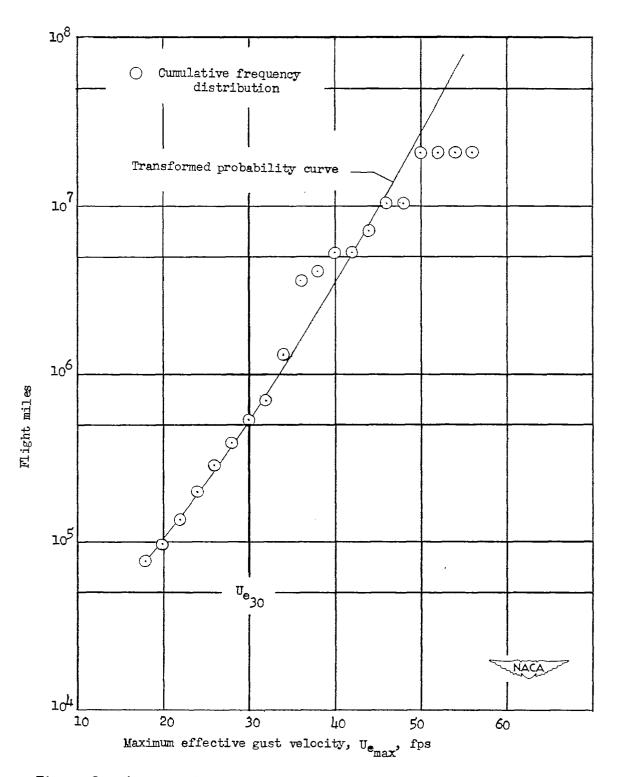


Figure 3.- Average flight miles required for a maximum positive and negative effective gust velocity to equal or exceed a given value.

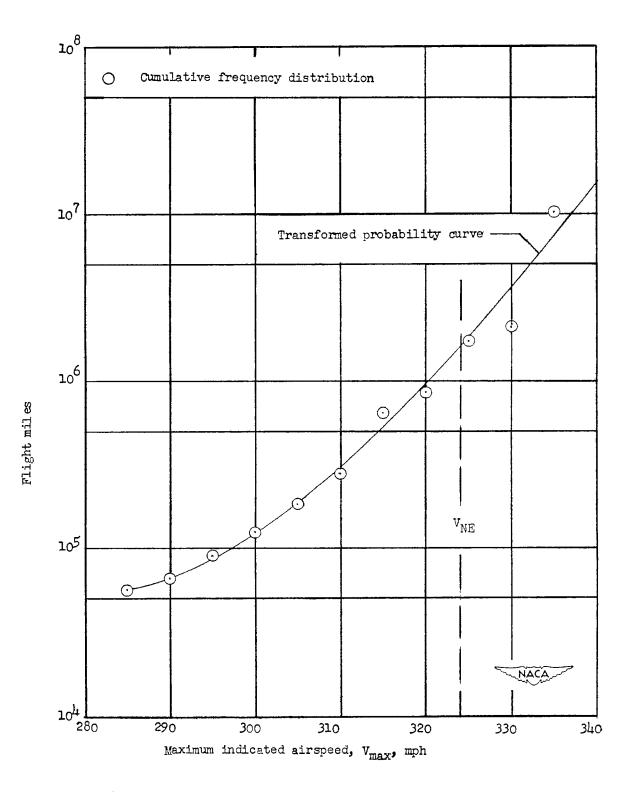


Figure 4.- Average flight miles required for the maximum indicated airspeed on a V-G record to equal or exceed a given value.

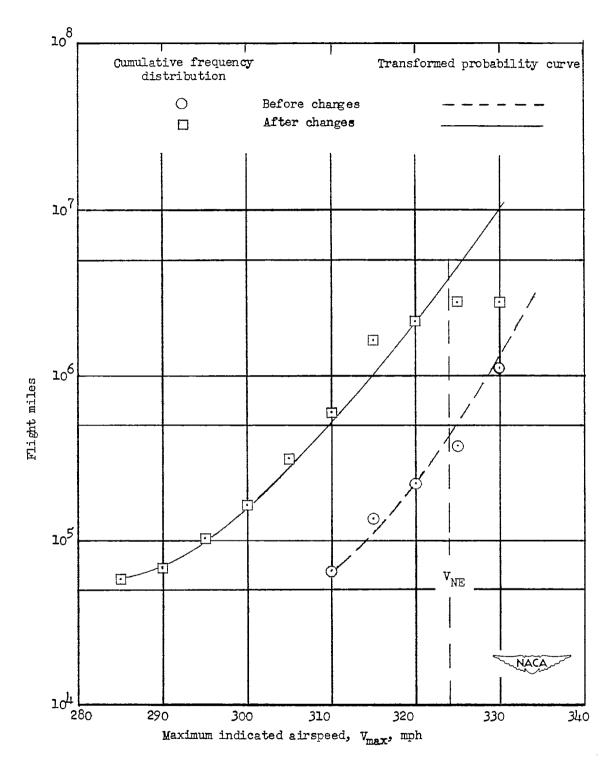


Figure 5.- Average flight miles required for the maximum indicated airspeed on a record to equal or exceed a given value for periods before and after changes in operating practice and airplane characteristics.

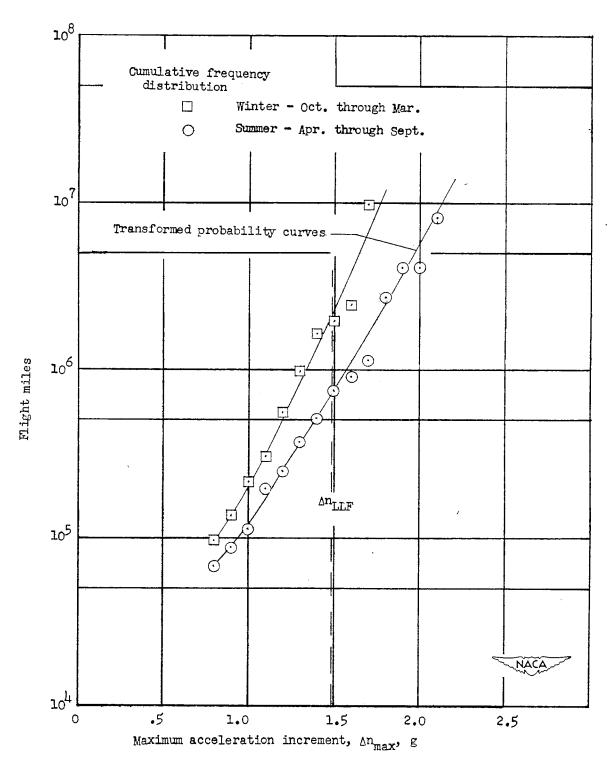


Figure 6.- Average flight miles required for a maximum positive and negative acceleration increment to equal or exceed a given value for summer and winter operations.

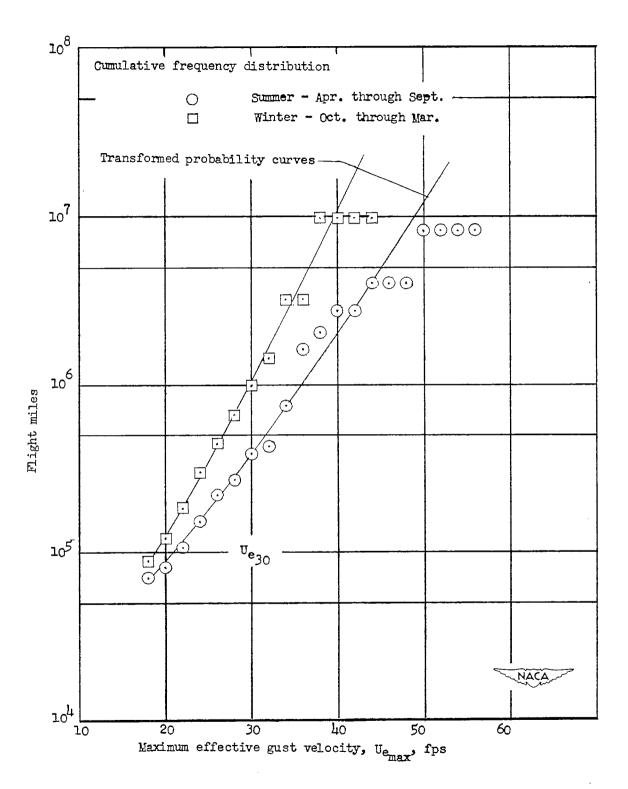


Figure 7.- Comparison of the average flight miles required to equal or exceed a given maximum effective gust velocity for summer and winter operations.

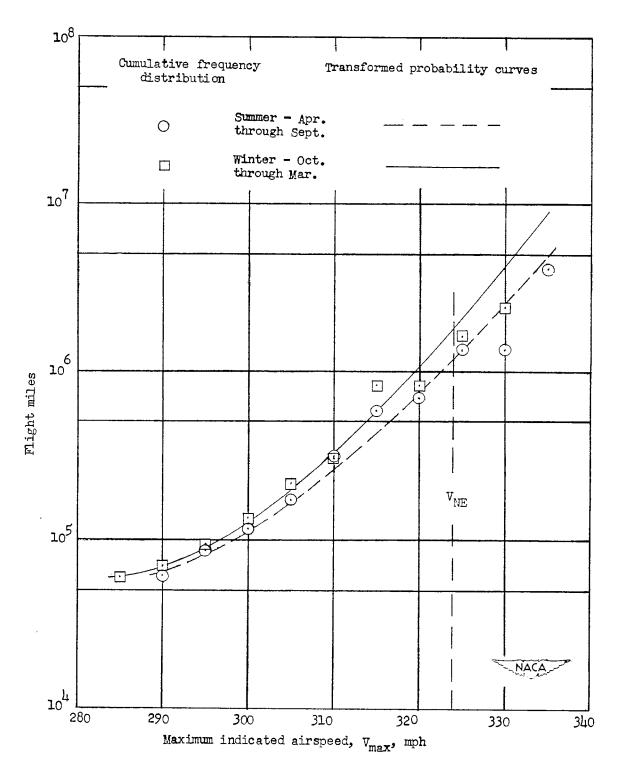


Figure 8.- Average flight miles required for the maximum indicated airspeed on a record to equal or exceed a given value for summer and winter operations.

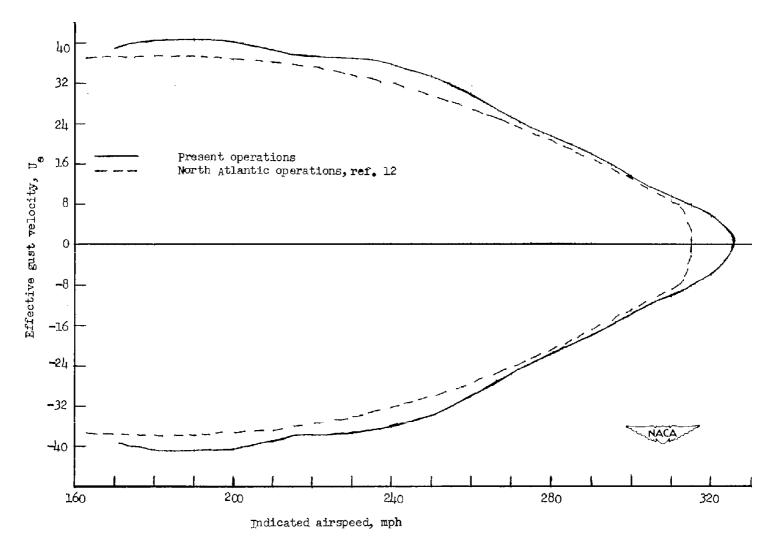


Figure 9.- Comparison of calculated flight envelopes of effective gust velocities for 10,000 hours of operations on domestic United States routes and British North Atlantic routes.